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Reducing nonideal to ideal coupling in random matrix description of chaotic scattering: Application to the time-delay problem

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We write explicitly a transformation of the scattering phases reducing the problem of quantum chaotic scattering for systems with M statistically equivalent channels at nonideal coupling to that for ideal coupling. Unfolding the phases by their local density leads to universality of their local fluctuations for large M. A relation between the partial time delays and diagonal matrix elements of the Wigner-Smith matrix is revealed for ideal coupling. This helped us in deriving the joint probability distribution of partial time delays and the distribution of the Wigner time delay.

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The random matrix theory (RMT) is generally accepted to be an adequate tool for describing various universal statistical properties of quantum systems with chaotic intrinsic dynamics; see Ref. [1] and references therein. In particular, one can distinguish two variants of the RMT approach allowing one to address the chaotic nature of quantum scattering. The first one [2] considers the scattering matrix S as the prime object without any reference to the system Hamiltonian. The probability distribution $P(S)$ of S at the fixed energy E of incident particles is chosen to satisfy a maximum entropy principle and natural constraints which follow from the unitarity and causality of S, and the presence (or absence) of the time-reversal (TRS) and spin-rotation (SRS) symmetries,

$$P(S)\propto\left|\frac{\det(1-\overline{S}^{\dagger}\overline{S})}{\det(1-\overline{S}^{\dagger}S)^2}\right|^{(\beta M+2-\beta)/2}. \qquad (1)$$

Such a distribution is known as the Poisson kernel [3] and uses the phenomenological average (or optical) S-matrix $\overline{S(E)}$ as the set of input parameters. Without loss of generality $\overline{S}$ can be considered as diagonal [4]. $P(S)$ depends also on the number of scattering channels M and the symmetry index β [$\beta=2$ for a system with broken TRS, and $\beta=1(4)$ if the TRS is preserved and the SRS is present (absent)].

The approach proved to be a success for extracting many characteristics important in the theory of mesoscopic transport [5]. However, correlation properties of the S-matrix at close values of energy E as well as spectral characteristics of an open system related to the so-called resonances turn out to be inaccessible in the framework of such an approach, essentially because of the one-energy nature of the latter. To address such quantities one needs to consider the Hamiltonian $\hat{H}$ of the quantum chaotic system as the prime building block of the theory. It amounts to treating $\hat{H}$ as a large $N\times N$ random matrix of appropriate symmetry and relating S to the Hamiltonian by means of standard tools of the scattering theory [6,7]. This idea supplemented with the supersymmetry technique of ensemble averaging [8] resulted in advance in calculating S-matrix correlation functions [6,9] and many other related characteristics such as, e.g., time delays [10–12]; see Refs. [11,1] for a review.

In the limit $N\to\infty$ one can prove [13] the equivalence of both mentioned approaches by deriving the Poisson kernel (1) from the Hamiltonian approach (see also Ref. [11]), with the average S-matrix being

$$\overline{S(E)}_{ab}=\frac{1-\gamma_a[iE/2+\pi\nu(E)]}{1+\gamma_a[iE/2+\pi\nu(E)]}\,\delta_{ab}, \qquad (2)$$

independent of β. Here, the average density of states $\nu(E)=\pi^{-1}\sqrt{1-(E/2)^2}$ determines the mean level spacing $\Delta=(\nu N)^{-1}$ of the closed system, and phenomenological constants $\gamma_c>0$ characterize the coupling strength to continuum in different scattering channels $(c=1,\ldots,M)$.

The particular case of ideal coupling, $\overline{S}=0$ [when the transmission coefficients equal unity for all channels; see Eq. (6) below], plays an especially important role for the S-matrix approach [14]. Equation (1) simplifies then to $P_0(S)=\text{const}$, which is invariant under the transformations of S, leaving the measure invariant. Such a situation corresponds to the so-called Dyson's circular ensemble (CE) of unitary matrices and is much simpler to handle analytically.

A general situation of nonideal coupling, $\overline{S}\neq 0$, turns out to be much more complicated. It is natural to expect, however, that results obtained for the case of nonideal coupling could be related to those at ideal coupling. Although many useful ideas around such a relation were discussed in the literature [2,13,11,15], we are not aware of explicit relations, to the best of our knowledge.

In this Rapid Communication we consider the most simple but physically important case of statistically equivalent channels. We demonstrate the validity of the following simple statement (and discuss several applications of it): Let $S(E)=U\hat{s}(E)U^{\dagger}$, where $\hat{s}(E)=\text{diag}(e^{2i\delta_1(E)},\ldots,e^{2i\delta_M(E)})$, be the random S-matrix at the energy E, the distribution of which is given by the Poisson kernel (1) with an explicit parameterization of $\overline{S(E)}$ from Eq. (2) ($\gamma_c = \gamma$ for all c). Then for every E the transformation of the eigenphases $\delta_c(E)$,

$$\phi_c = \arctan\left[\frac{1}{\pi\nu(E)}\left(\frac{1}{\gamma}\tan\delta_c(E) + \frac{E}{2}\right)\right] \tag{3}$$

maps them to the eigenphases ϕ_c of the random scattering matrix, the distribution of which is given by the CE of the same symmetry. In particular, the joint probability density function (JPDF) of ϕ_c is [1]

$$p_0(\{\phi_c\}) \propto \prod_{a<b} |e^{2i\phi_a} - e^{2i\phi_b}|^\beta. \tag{4}$$

The matrix U of energy dependent eigenvectors uniformly distributed in the orthogonal, unitary, or symplectic group (for $\beta = 1,2$, or 4, respectively) is not affected by map (3). This is a consequence of statistical equivalence of the scattering channels.

The suggested transformation was first noticed and verified in Ref. [11] for the case of broken TRS ($\beta = 2$) and further exploited in Ref. [12]. It can be easily generalized to the other symmetry classes as follows. We calculate first the Jacobian of the transformation (3). After a simple algebra it can be represented as

$$\frac{\partial\phi_c}{\partial\delta_c} = \frac{\cos^2\phi_c}{\pi\nu\gamma\cos^2\delta_c} = \frac{T}{|1 - \overline{S}^* e^{2i\delta_c}|^2}, \tag{5}$$

where

$$T(E) = 1 - |\overline{S(E)}|^2 = \frac{4\gamma\,\pi\nu(E)}{1 + \gamma^2 + 2\gamma\,\pi\nu(E)} \tag{6}$$

is the energy dependent transmission coefficient [6]. We note that exactly the same factors as Eq. (5) appear in Eq. (1). Employing the identity

$$|e^{2i\phi_a} - e^{2i\phi_b}| = |e^{2i\delta_a} - e^{2i\delta_b}|\left|\frac{\cos\phi_a\cos\phi_b}{\pi\nu\gamma\cos\delta_a\cos\delta_b}\right|, \tag{7}$$

we substitute Jacobians (5) into Eq. (4) and, making use of the identity $\Pi_{a<b}^M f_a f_b = (\Pi_c f_c)^{M-1}$, arrive at

$$p(\{\delta_c\}) \propto \prod_{a<b} |e^{2i\delta_a} - e^{2i\delta_b}|^\beta \prod_{c=1}^{M} \left|\frac{\partial\phi_c}{\partial\delta_c}\right|^{\beta(M-1)/2+1}. \tag{8}$$

With Eq. (5) taken into account, we immediately recognize in this expression the JPDF of the eigenphases corresponding to Poisson kernel (1). Due to the scalar nature of transformation (3) it does not change the matrix U of eigenvectors.

Let us start with considering the mean density $\rho(\delta)$ of scattering (eigen)phases at arbitrary coupling. It is self-evident that the phases in the CE (i.e., for the case $\overline{S} = 0$) are uniformly distributed on the unit circle, the average density being merely $\rho_0(\phi) = (1/M)\Sigma_c \overline{\delta(\phi - \phi_c)} = 1/\pi$. The corresponding density for $\overline{S} \neq 0$ is not constant. Indeed, using the identity $\rho(\delta)d\delta = \rho_0(\phi)d\phi$, we see that

$$\rho(\delta) = \frac{1}{\pi}\left|\frac{\partial\phi}{\partial\delta}\right| = \frac{T}{\pi|1 - \overline{S}^* e^{2i\delta}|^2}. \tag{9}$$

Although simple, this relation is an important one and establishes the physical meaning of the Jacobians of transformation (3) relating them to the corresponding densities of the scattering phases. Density (9), being expressed in terms of $\overline{S}$ only, does not depend on the particular choice of $\overline{S}$ used in the derivation as long as the average S-matrix is proportional to the unit matrix.

It is instructive to look at Eq. (8) in the limit of large number of channels when the typical difference $\delta_a - \delta_b \sim 1/M \ll 1$. Then one can expand $\delta_c = \delta_0 + \tilde{\delta}_c$ ($\tilde{\delta}_c \ll 1$) around, say, δ_0. The leading contribution is given by $p(\{\tilde{\delta}_c\}) \propto \Pi_{a<b}|\tilde{\delta}_a - \tilde{\delta}_b|^\beta \Pi_c |\partial\phi_c/\partial\delta_c|_{\delta_0}^{\beta(M-1)/2+1}$, which further goes to $p_0(\{\tilde{\phi}_c\}) \propto \Pi_{a<b}|\tilde{\phi}_a - \tilde{\phi}_b|^\beta$ and agrees with distribution (4) of the CE upon the proper rescaling of the phases,

$$\tilde{\phi}_c = |\partial\phi_c/\partial\delta_c|_{\delta_0}\tilde{\delta}_c = \pi\rho(\delta_0)\tilde{\delta}_c. \tag{10}$$

We see that in the limit $M \gg 1$ the local fluctuations of the phases unfolded by their local density turn out to be uniformly described by the CE at arbitrary coupling strength. Such a universality in statistics of phases of random unitary (scattering) matrices has much in common with that typical for eigenvalues of random Hamiltonian matrices [1] and is in agreement with results of realistic numerical simulations for $M = 23$ [16].

Let us now consider an application of the same ideas to the time-delay problem, where such a universality reveals itself explicitly. Following the original wave-packet analysis by Wigner and Smith [17] it is natural to define [11] the *partial* time delays via the energy derivative of the scattering phases, $\tau_c = 2\hbar\,\partial\delta_c/\partial E$. Their statistical properties have been studied in much detail in the framework of the Hamiltonian approach for the case of broken [11] and preserved TRS as well as in the whole crossover region of gradually broken TRS [12]. Recently, some of these predictions were successfully verified with the model of a quantum Bloch particle chaotically moving in a superposition of ac and dc fields [18].

In particular, the mean density of partial time delays $\mathcal{P}(\tau) = (1/M)\Sigma_c \overline{\delta(\tau - \tau_c)}$ turns out to be especially simple at ideal coupling, $T = 1$, when it reads as

$$\mathcal{P}_0(t = \tau/t_H) = \frac{(\beta/2)^{\beta M/2}}{\Gamma(\beta M/2)} \frac{e^{-\beta/2t}}{t^{\beta M/2+2}}, \tag{11}$$

with $t_H = 2\pi\hbar/\Delta$ being the Heisenberg time. Due to Eq. (10), the partial time delays at ideal and nonideal coupling ($\tau_c^{(0)}$ and τ_c, respectively) are simply related as

$$\tau_c^{(0)} = 2\hbar\,\partial\phi_c/\partial E = \pi\rho(\delta_c)\tau_c. \tag{12}$$

Here, we have neglected the smooth nonresonant dependence of $\rho(\delta)$ on E. Since the phase and its derivative (the partial time delay) are uncorrelated quantities in the CE [19], their joint distribution factorizes: $\hat{p}_0(\phi,\tau^{(0)}) = (1/\pi)\mathcal{P}_0(\tau^{(0)})$. This is not the case for $\hat{p}(\delta,\tau)$, when $\overline{S} \neq 0$. The relation $\hat{p}_0(\phi,\tau^{(0)})d\phi\, d\tau^{(0)} = \hat{p}(\delta,\tau)d\delta\, d\tau$ between them allows us, however, to represent the density of partial time delays at nonideal coupling as

$$\mathcal{P}(\tau) = \int_0^\pi d\delta \left|\frac{\partial(\phi,\tau^{(0)})}{\partial(\delta,\tau)}\right| \hat{p}_0(\phi(\delta),\tau^{(0)}(\delta,\tau)) = \int_0^\pi \frac{d\delta}{\pi}[\pi\rho(\delta)]^2\, \mathcal{P}_0(\pi\rho(\delta)\tau). \tag{13}$$

One can easily convince oneself [20] that such a formula reproduces in every detail the expression obtained in Ref. [11] by means of supersymmetry calculations. It is worth mentioning that the density of phases (9) is independent of the underlying symmetry, and therefore Eq. (13) is also valid for the crossover regime of partly broken TRS. [Note that in the crossover regime $\mathcal{P}_0(t)$ is a slightly more complicated function; see Ref. [12]].

Expression (13) is the proper one for generalization to the JPDF of the partial time delays, $w(\{\tau_c\})$. Before doing this, we first establish a useful relation between τ_c and the matrix elements of the Wigner-Smith time-delay matrix $Q = -i\hbar(\partial S/\partial E)S^\dagger$ [17]. Writing S in the eigenbasis representation as $S = U\hat{s}U^\dagger$, one obtains

$$U^\dagger Q U = -i\hbar\frac{\partial \hat{s}}{\partial E}\hat{s}^\dagger + i\hbar\left[\hat{s}, U^\dagger\frac{\partial U}{\partial E}\right]\hat{s}^\dagger, \tag{14}$$

where [,] denotes the commutator. The matrix $\hat{s}$ being diagonal, the diagonal elements of the second term in Eq. (14) are zero, whereas the first term is exactly the diagonal matrix of the partial time delays. Thus, the partial time delays coincide with the diagonal elements of the time-delay matrix taken in the eigenbasis of the scattering matrix,

$$\tau_c = [U^\dagger Q U]_{cc}. \tag{15}$$

The physical meaning of the diagonal elements of the time-delay matrix is well known: they describe the time delay of a wave-packet incident in a given channel [17,21]. Thus, relation (14) sheds more light on the physical meaning of the somewhat formally defined partial time delays. In particular, one expects that for the case of ideal coupling the inherent rotational invariance of the problem makes all the basises statistically equivalent, and thus, the JPDF of diagonal elements of the Q-matrix should coincide with that of partial time delays.

The latter claim can be substantiated as follows. Following the insightful paper [19], it is convenient to consider the ‘‘symmetrized’’ time-delay matrix Q_s,

$$Q_s = S^{-1/2}QS^{1/2} = -i\hbar S^{-1/2}\frac{\partial S}{\partial E}S^{-1/2}. \tag{16}$$

This similarity transformation unveils the symmetry that is hidden in Q: Q_s is already a real symmetric (Hermitian, or quaternion self-dual) matrix for $\beta=1$ (2, or 4). In the eigenbasis of S the diagonal elements of Q_s and those of Q coincide. Moreover, in the case of chaotic scattering with ideal coupling, the matrix Q_s turns out to be statistically independent of S, their joint probability density being $\hat{P}_0(S,Q_s) = P_0(S)W_0(Q_s)$, where

$$W_0(Q_s) \propto \theta(Q_s)\det(Q_s)^{-3\beta M/2-2+\beta}e^{-(\beta/2)t_H \mathrm{tr}\, Q_s^{-1}} \tag{17}$$

is the probability density of the time-delay matrix [19]. The latter is manifestly invariant under the choice of the basis for Q_s, proving the above statement on the relation between statistics of partial time delays and diagonal elements of the Wigner-Smith matrix.

To find the corresponding JPDF $w_0(\{\tau_c\})$ one has to integrate out all off-diagonal elements of Q_s, which is a hard problem in general. For the case of unitary symmetry, $\beta = 2$, one can perform the job by splitting the integration into that over the matrix $\hat{q} = \mathrm{diag}(q_1, \ldots, q_M)$ of eigenvalues of Q_s and that of the eigenvectors, V,

$$w_0^u(\{\tau_c\}) \propto \int d[\hat{q}]\frac{\theta(\hat{q})\Delta^2(\hat{q})}{\det(\hat{q})^{3M}}e^{-t_H \mathrm{tr}\,\hat{q}^{-1}}\mathcal{Q}(\{\tau_c\}), \tag{18}$$

with $\Delta(\hat{q}) = \Pi_{a<b}(q_a - q_b)$ being the Vandermonde determinant. Here $\mathcal{Q}(\{\tau_c\}) = \int d[V]\Pi_{c=1}^M \delta(\tau_c - (V\hat{q}V^\dagger)_{cc})$ stands for the remaining integral over the unitary group which can be done, following Ref. [22], by means of the famous Itzykson-Zuber formula [23]. Finally, we find it more convenient to define the generating function of partial time delays rather than the JPDF itself and obtain

$$\langle e^{-i(k_1\tau_1 + \ldots + k_M\tau_M)}\rangle_\tau \propto \frac{\det[\psi_j(k_l)]}{\prod_{a<b}(k_a - k_b)}, \tag{19}$$

where $\psi_j(k_l) = \int_0^\infty dq\, q^{j-3M}e^{-ik_l q - t_H/q}$, the index l spans the values $l = 1, \ldots, M$, and $j = 0,1,\ldots,M-1$.

Such an expression allows us to calculate all the moments and correlation functions of partial time delays by a simple differentiation. Moreover, setting in the preceding equation $k_1 = \cdots = k_M = k$ and calculating the corresponding limit in the right-hand side, we come to a convenient representation for the distribution $\mathcal{P}_M^u(t_w)$ of the Wigner time delay, $t_w = (\tau_1 + \ldots + \tau_M)/Mt_H$, for a system with broken TRS and ideal coupling to continuum,

$$\mathcal{P}_M^u(t_w) \propto \int_{-\infty}^{\infty} dk e^{iMkt_w}\det[\psi_j^{(n)}(k)], \tag{20}$$

where $\psi_j^{(n)}(k) \equiv d^n\psi_j(k)/dk^n$, and $j,n = 0,\ldots,M-1$.

The distribution of the Wigner time delay was earlier calculated explicitly only for the case of $M=1$ [11,12,24], when it follows from Eq. (11). Compact expression (20) is valid for $\beta=2$ and arbitrary M [25]. For $M=2$, Eq. (20) can be integrated further to yield

$$\mathcal{P}_2(t_w) \propto t_w^{-3(\beta+1)} e^{-\beta/t_w} U\left(\frac{\beta+1}{2}, 2\beta+2, \frac{\beta}{t_w}\right), \quad (21)$$

with $U(a,b,z)=[1/\Gamma(a)]\int_0^\infty dy\, y^{a-1}(1+y)^{b-a-1}e^{-zy}$ being the confluent hypergeometrical function. Here we represented the above distribution (21) in a form covering all $\beta=1,2,4$, which will be verified below. In particular, the asymptotic behavior at $t_w \gg 1$ is $\mathcal{P}_2(t_w) \propto t_w^{-\beta-2}$, in agreement with the known universal tail $t^{-\beta M/2-2}$, which is typical for the time-delay distributions in open chaotic systems [11,12,18,19].

To verify Eq. (21) for $\beta=1,4$, it is convenient to consider a general problem of finding the distribution $\tilde{W}_0(\tilde{Q})$ of the $n\times n$ submatrix $\tilde{Q}$ standing on the main diagonal of Q_s. This distribution is found to be

$$\tilde{W}_0(\tilde{Q}) \propto \theta(\tilde{Q}) \det(\tilde{Q})^{-\beta(M/2+n-1)-2} e^{-\beta t_H \mathrm{tr}\, \tilde{Q}^{-1}/2}. \quad (22)$$

The particular case $n=1$ reproduces result (11) of the Hamiltonian approach. Equation (22) for $n=2$ helps in calculating the joint distribution $\hat{w}_0(t_1,t_2)$ of two partial time delays $t_{1,2}=\tau_{1,2}/t_H$ for arbitrary M. One obtains

$$\frac{\hat{w}_0(t_1,t_2)}{\mathcal{P}_0(t_1)\mathcal{P}_0(t_2)} \propto \frac{U\left(\frac{\beta}{2}, \frac{\beta M}{2}+\beta+2, \frac{\beta}{2t_1}+\frac{\beta}{2t_2}\right)}{(t_1 t_2)^{\beta/2}}. \quad (23)$$

The knowledge of $\hat{w}_0(t_1,t_2)$ allows us to find further the distribution of the Wigner time delay for $M=2$ and thus prove formula (21) for any β. As follows from Eq. (23), there exist nonvanishing correlations between the partial time delays. They are, however, of different nature as compared to the correlations between the *proper* time delays (the eigenvalues of Q) which show repulsion [19].

For $\overline{S}\neq 0$ the matrices S and Q_s cease to be statistically independent variables and do correlate. Therefore, statistical properties of diagonal elements of Q in arbitrary basis (save the eigenbasis of S) are different from that of partial time delays, unless coupling is ideal. Still, the JPDF $w(\{\tau_c\})$ of the partial time delays at nonideal coupling can be found by repeating basically the same steps which lead to Eq. (13). The identity $\hat{p}(\{\delta_c\},\{\tau_c\})d[\delta]\,d[\tau] = \hat{p}_0(\{\phi_c\},\{\tau_c^{(0)}\})d[\phi]\,d[\tau^{(0)}]$, together with the statistical independence of ϕ_c and $\tau_c^{(0)}$ [which follows from Eq. (17)], allows us to relate $w(\{\tau_c\})$ and $w_0(\{\tau_c\})$ as follows:

$$w(\{\tau_c\}) = \int d[\delta] \left| \frac{\partial(\{\phi_c\},\{\tau_c^{(0)}\})}{\partial(\{\delta_c\},\{\tau_c\})} \right| \hat{p}_0(\{\phi_c\},\{\tau_c^{(0)}\})$$
$$= \int d[\delta] \prod_{c=1}^{M} [\pi\rho(\delta_c)]\, p(\{\delta_c\}) w_0(\{\pi\rho(\delta_c)\tau_c\}), \quad (24)$$

where $d[\delta]$ means the product of differentials.

In conclusion, we suggest the transformation of the scattering phases, allowing one to reduce the problem of quantum chaotic scattering with statistically equivalent channels at arbitrary coupling to that for ideal coupling. Applications of this transformation to statistical properties of phases and those of time delays are discussed.

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[1] T. Guhr *et al.*, Phys. Rep. **299**, 189 (1998).
[2] P.A. Mello *et al.*, Ann. Phys. (N.Y.) **161**, 254 (1985); W.A. Friedman and P.A. Mello, *ibid.* **161**, 276 (1985).
[3] L.K. Hua, *Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains* (AMS, Providence, 1963).
[4] C.A. Engelbrecht and H.A. Weidenmüller, Phys. Rev. C **8**, 859 (1973).
[5] C.W.J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997).
[6] J.J.M. Verbaarschot *et al.*, Phys. Rep. **129**, 367 (1985).
[7] V.V. Sokolov and V.G. Zelevinsky, Phys. Lett. B **202**, 140 (1988); Nucl. Phys. A **504**, 562 (1989).
[8] K.B. Efetov, Adv. Phys. **32**, 53 (1983).
[9] N. Lehmann *et al.*, Nucl. Phys. A **582**, 223 (1995).
[10] N. Lehmann *et al.*, Physica D **86**, 572 (1995).
[11] Y.V. Fyodorov and H.-J. Sommers, J. Math. Phys. **38**, 1918 (1997).
[12] Y.V. Fyodorov *et al.*, Phys. Rev. E **55**, R4857 (1997).
[13] P.W. Brouwer, Phys. Rev. B **51**, 16 878 (1995).
[14] It is worth stressing that $\overline{S}=0$ requires both $E=0$ and all $\gamma_c=1$, according to Eq. (2).
[15] V.A. Gopar and P.A. Mello, Europhys. Lett. **42**, 131 (1998).
[16] B. Dietz *et al.*, Phys. Lett. A **215**, 181 (1996).
[17] E.P. Wigner, Phys. Rev. **98**, 145 (1955); F.T. Smith, Phys. Rev. **118**, 349 (1960).
[18] M. Glück *et al.*, Phys. Rev. E **60**, 247 (1999).
[19] P.W. Brouwer *et al.*, Phys. Rev. Lett. **78**, 4737 (1997); Waves Random Media **9**, 91 (1999).
[20] For convenience of the reader we note that the identity $\int_0^{2\pi} d\delta\, f(p\cos\delta+q\sin\delta)=2\int_0^{\pi}d\delta\, f(\sqrt{p^2+q^2}\cos\delta)$ allows us to write Eq. (13) in the form of Refs. [11,12], with $\pi\rho(\delta)$ being replaced by $\pi\tilde{\rho}(\delta)=[g-\sqrt{g^2-1}\cos\delta]^{-1}$, $g=2/T-1$.
[21] V.L. Lyuboshitz, Phys. Lett. B **72**, 41 (1977); Sov. J. Nucl. Phys. **27**, 502 (1978); Pis'ma Zh. Éksp. Teor. Fiz. **28**, 32 (1978) [JETP Lett. **28**, 30 (1978)].
[22] Y.V. Fyodorov and B.A. Khoruzhenko, Phys. Rev. Lett. **83**, 65 (1999).
[23] C. Itzykson and J.B. Zuber, J. Math. Phys. **21**, 411 (1980).
[24] V.A. Gopar *et al.*, Phys. Rev. Lett. **77**, 3005 (1996).
[25] We note, that due to the relation $t_w=\mathrm{tr}\, Q_s/Mt_H$, Eq. (20) can be also derived directly from Eq. (17).